War, mathematics, and mathematicians

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1. Introduction

On the surface, mathematics and warfare have little in common. Mathematics is, if anything, a purely intellectual pursuit, while war is a highly practical method of applying violence to achieve political or economic advantage or resisting such attempts by the adversary. Nevertheless, war and mathematics have their points of tangency. Mathematics exists for two reasons. One is *l'art pour l' art*; the other is the astonishing usefulness of mathematics to all kinds of applications. Not seldom does a mathematical result or method come into existence long before its need for some useful purpose is realized. Cases exist where that need appears in military circumstances.

I was personally reminded of the connections of mathematics and war when teaching courses on the history of mathematics. It seems that a remarkably large number of outstanding mathematicians, starting in the 18th century—people who have left their mark in the history books—have at one or another point in their career served as mathematics teachers in war schools.

This paper gives a rather rhapsodic report of these reflections. My point of view is rather historically oriented. This is partly due to my starting point. In modern technology-oriented warfare, mathematics is very much integrated into weapons, communications, logistics, and other systems. Its presence is not always easily perceived, and direct knowledge of the mathematics involved in a particular system is not necessary for applying it. It is commonly said, and true, that mathematics pervades modern military technology. It is equally true that the math involved seldom is visible to the users of the technology, the soldiers and military leaders. It is not my aim to point out or clarify the mathematical ideas residing in, say, weapon and communication systems. Rather, I try to offer some glimpses at important mathematicians of different ages with connections to war. In the conclusion, I try to vaguely establish a wisdom based on these glimpses. All technicalities are avoided.

The material mentioned here on mathematicians can be found in most general histories of the subject, such as Carl Boyer's work (1985); thus, specific references are not given. The material in the wonderful MacTutor History of Mathematics Archive maintained by St. Andrews University in Fife, Scotland, at http://www-groups.dcs.st-and.ac.uk/~history, has been used.

2. Antiquity

Early connections of warfare and mathematics-oriented activities can be seen in counting troops and sharing the bounty. Good examples are found in the Bible—for instance, in the elaborate bounty division computations of Chapter 31 of the Book of Numbers. But to see mathematicians in the stricter sense involved with war, we have to look at classical Greece.

2.1. Archimedes of Syracuse

One of the great tales of Antiquity is about science and war. The main character is **Archimedes** (287–212 BC), and the site is Syracuse. According to the legend, mainly coming to us via Plutarch, Archimedes devised a number of miraculous defense devices to aid his friend Hieron, the tyrant of Syracuse, against the Romans who besieged the city in 212 BC in the second Punic

War. At that time, Archimedes was over 70 years of age. The legend, as told by Plutarch in his *Life of Marcellus*, also notes that Archimedes did not really think that the mechanical and optical aids to war that he invented were of importance when compared to the abstract science of geometry, which he enjoyed pursuing. This is emphasized in the final act of the legend, the death of Archimedes at the sword of an ignorant Roman soldier when the old man was concentrating on a mathematical problem.

There is much to be doubted in the details of the legend of Archimedes and the siege of Syracuse, yet the main features are quite plausible, and they illustrate the broad lines of interaction of mathematics and warfare. Archimedes possessed an amount of knowledge of mathematics and physics unparalleled in his time (and for many centuries to come). His mind was not set to work on the practical problems of attack and defense, but, when needed, he was able to extract applicable knowledge that, for some time at least, made the defender superior.

Although we admire Archimedes, and with good reason, we must acknowledge that his inventions did not come from nowhere. The catapult was invented in around 400 BC, and literature as well as practical knowledge on its use and on the demands catapults imposed on fortifications, existed in his time (Cuomo 2001).

2.2. Apollonius of Perga

Another great mathematician of Antiquity whose indirect influence in military matters is great is **Apollonius of Perga** (262–190 BC). Not that he knew it. The work of Apollonius is a good example of roads of invention that are not always direct. The masterwork of Apollonius described the properties of conic sections—i.e., ellipses, parabolas, and hyperbolas (names coined by him)—which very much later, starting with the observations and inventions of Kepler and Newton, were seen to have a variety of meanings in military contexts. The trajectory of a projectile in vacuum and a constant gravity field (the first approximation of the flight of a bullet) is a parabola, the trajectory of a body moving unconstrained in a centrally symmetric gravity field (a ballistic missile or satellite) is an ellipse, and hyperbolas are seen in such contexts as voice reconnaissance and satellite positioning systems. Undoubtedly, the results of Apollonius would have been rediscovered in time with methods more modern than he possessed, but there was no need, since they already were there.

3. From the Renaissance to the French Revolution

3.1. Niccolo Tartaglia

The Italian **Niccolo Tartaglia** (1500–1557) was affected by war in his youth: born Fontana, he obtained the name Tartaglia, meaning "Stammerer," as a result of a speech defect caused by the wounds he got in the sack of Brescia, his home town, by the French in 1512. Tartaglia's name is connected with the most sensational mathematical innovation of the Renaissance, and probably the most colorful story in the history of mathematics, the algebraic solution of equations of third degree. But Tartaglia was also active in the emerging art of artillery. He was consulted on the maximal range of a cannon, and he published two volumes, *La Nova Scientia* (1537) and *Questioni et Inventioni Diverse* (1546), on the subject. Newtonian mechanics were unknown in Tartaglia's time. Instead of an exact solution, he used modeling. His approximation of the trajectory of a projectile comprised three parts: a linear rising part where the action of gunpowder is felt, a curved part in which the projectile turns back toward earth, and a final linear part at the end of which the projectile hits the ground. It was Tartaglia's intention, announced in both books, to publish firing tables connecting gun elevation and range, but neither book actually contained such information. With the information available to Tartaglia, constructing such tables would have been impossible.

3.2. François Viète

François Viète (1540–1603) is one of the great names in the history of mathematical notation and formalism: he is one of the initiators of the modern notation of decimal fractions and can be considered the father of algebra in the sense that he introduced the use of letters to represent numbers in algebraic formulae. Viète was a lawyer by profession and served as a counselor to Henry of Navarre, later Henry IV. In this role he was an early representative of another connection of mathematics and the military, reconnaissance and cryptography. Henry of Navarre and Philip II of Spain were at one time contesting the French throne, and the former's forces intercepted a coded message containing Philip's military plans. The code was eventually cracked by Viète, and the leak of information so surprised Philip that he complained to Henry of unfair conduct (i.e., use of black magic).

3.3. Leonhard Euler

The Swiss-born Leonhard Euler (1707–1783) is perhaps the most prolific mathematician of all time. It is noteworthy that he first earned a reputation in the world of science with a work closely related to naval warfare, a contribution on the balancing of a ship and placement of its masts, submitted to the French Academy in response to a competition question (bear in mind that Euler came from a land-locked country and had never seen open sea). Euler later moved to Russia to assume a post at the newly created St. Petersburg Academy, where his multiple duties included teaching cadets—this gave him a substantial part of his income—as well as various consulting jobs for the Russian navy.

3.4. Gaspard Monge and military schools

The emergence of firearms transformed warfare of the 16th century onwards. One of the consequences was the need for a different kind of fortifications. French engineer **Maréchal de Vauban** (1633–1707) was instrumental in this change. His system of fortifications very much relied on geometric principles assuring both good resistance to cannon fire and free use of one's own fire. Another consequence was a need for officers who were capable of not only leading their men—something a nobleman raised among servants could do without special education—but also understanding the use of artillery, mines, and fortification. The 18th century was the starting point for many institutions of higher military education. One of these establishments was the École Royale du Génie at Mézières. It was established in 1748 to educate young noblemen as engineers for the French forces. **Gaspard Monge** (1746–1818) joined the institution in 1765 as a draftsman. A profound geometer, he soon developed a superior method for drawing plans of fortresses. This method, descriptive geometry, was for a time classified military information. Later it became one of the universal backbones of technical education. Monge as a mathematician is considered the father of differential geometry, the study of geometrical relations via tools from mathematical analysis.

Monge was active in the French revolution, serving as a cabinet minister. He was instrumental in setting up the École polytechnique in 1794. Remaining one of the *Grandes écoles* of France and a military institution until 1970, it, with its excellent teachers and their textbooks, has had a remarkable influence on the development of mathematics, especially in the first half of the 19th century. Also, the emerging higher technical education eventually leading to the modern technical universities got its models from the school whose foundations were laid by Monge.

The influence of the École polytechnique can be seen in the development of the US Military Academy at West Point. After a slow start in 1802, the development of the academy gained momentum in 1812 with the creation of three professorships—in natural and experimental philosophy, in mathematics, and in the science of engineering. One of the first acts of the institution's superintendent, **Sylvanus Thayer** (1785–1872), the "Father of West Point," was to sail to Europe to obtain books and instruments. Thayer's main contact point was the École, whose system he tried to reproduce at West Point (Albree et al. 1991).

Finnish officer education, generally considered to have started with the Haapaniemi officers' school in 1779, was not mathematically inclined. Nevertheless, mathematics had its place in the curriculum, and later, when the school was in the town of Hamina, it had some noteworthy mathematics teachers, among them Major General **Edvard Engelbert Neovius** (1823–1888), the grandfather of famous mathematicians Rolf and Frithiof Nevanlinna.

3.5. Mathematicians at the time of the French Revolution

Monge was not the only French mathematician with military connections at the time of the Revolution. At least four names of the highest rank in the hall of fame of mathematics were connected with the military even before that time. **Pierre-Simon Laplace** (1749–1827) and *Adrien-Marie Legendre* (1752–1833) started their careers as teachers at the École Militaire in Paris in the 1770s; *Joseph-Louis Lagrange* (1736–1813) was a teacher at the Regie Scuole Teoriche et Pratiche di Artiglieria e Fortificazione, established in 1739 in Turin, in the Kingdom of Savoy; and *Joseph Fourier* (1768–1830) both studied and taught at the École Royale Militaire of Auxerre. A number of educational institutions run primarily by the Church were turned by Louis XVI into royal military schools in 1776, the one at Auxerre among them.

Laplace, a pioneer of mathematical analysis and probability theory, can claim a lasting place in the military applications of mathematics through his method of transforming differential equations into algebraic ones, a standard tool in control theory—for instance, in the design of missile guidance systems. Legendre made his scientific breakthrough by winning a 1782 prize contest arranged by the Berlin Academy of Sciences on the topic of determining the trajectory of a projectile while taking into account the variation of air pressure at different altitudes; Legendre's pioneering work on the subject was titled *Recherches sur la trajectoire des projectiles dans les milieux résistants.* Fourier's name is forever preserved in Fourier analysis and the Fourier transform, the fundamental mathematical tool in signal analysis and thus essential in electronic communications and electronic warfare. Fourier arrived at his revolutionary idea of constructing a function by adding harmonic vibrations of varying frequency not in considering wave phenomena but through analysis of the propagation of heat. Thus Fourier's results are another example of a highly useful method developed in a quite different environment.

The French Revolution produced one of the most successful (most of the time) military leaders, **Napoleon Bonaparte**. It is interesting that he is probably the only person of high military credentials who also has a mathematical theorem named after him: the Theorem of Napoleon states that the centers of the equilateral triangles drawn on the sides of an arbitrary triangle always are vertices of an equilateral triangle. No clear evidence exists to show that the theorem was actually invented by the French emperor, but Napoleon's interest in mathematics is well known. Two of his most important scientific assistants on the Egyptian Expedition in 1798–99 were Monge and Fourier. The French even had time to establish an academy on this campaign, l'Institut d'Egypte. It had a mathematics division, and among its 12 members were Monge, Fourier, and Napoleon himself.

3.6. Soldiers turned mathematicians

The 19th century saw some instances of profound mathematical inventions by people trained for an officer's career. **Jean-Victor Poncelet** (1788–1867) was an engineer in the French force invading Russia in 1812. He was wounded and imprisoned at Krasnoy, near Smolensk. While in war prison in Russia, Poncelet developed ideas leading to a whole new branch of geometry, projective geometry. The Hungarian **Janos Bolyai** (1802–1860), one of the inventors of non-Euclidean geometry, of revolutionary importance to the emergence of modern abstract mathematics, was also trained as an officer, in the engineering corps of the Austrian Empire. To him, the officer's career was a substitute for mathematics, a profession he and his family could not afford. Bolyai was reputedly the best swordsman and dancer in the Austrian Imperial Army.

4. The world wars

4.1. Operations research

Frederick William Lanchester (1868–1946) was not a mathematician. His main achievements are in the fields of automotive and flight technology. During World War I, his attention turned to air warfare. He observed that a battle can be given a simple mathematical model relating the attrition of the adversaries' forces by a pair of differential equations parameterized by effectiveness factors for both sides. Lanchester's observations have led to much, still active research in battle modeling.

Lanchester is often viewed as a pioneer in the science of operations research, the core of which is the application of mathematically oriented methods to questions that also have human aspects, often decisions. Military operations are a good example of such questions. The Second World War actually gave birth to operations research in the modern sense. During this prolonged and extremely many-faceted conflict, many of the participants, notably the British and the Americans, mobilized much mathematical and scientific talent to analyze and develop various procedures for fighting and operations needed to sustain the fighting. After the war, much of the know-how was recruited with the personnel by the private sector to be of use in the ongoing non-bellicose conflicts of business.

Study and modeling of conflict also gave rise to a mathematical theory, game theory. This, too, began as a mostly intellectual adventure, of one of the most versatile and influential mathematicians of the 20th century, **John von Neumann** (1903–1957). The Hungarian-American von Neumann's contributions to the computational and organizational tasks in the Manhattan Project, leading to the ultimate link of science and war, the atomic weapon, were in many ways decisive, too.

4.2. Cryptography

The science and practice of cryptography has ancient roots and is closely tied to warfare. One of the simpler ways to try to conceal information is named after Julius Caesar. A comprehensive treatment of the connections of cryptology, mathematics, and war is beyond the scope of this review. During the Second World War, a number of mathematicians in different countries, Finland included, were employed for cryptological purposes. By far the most famous of them is the Briton **Alan M. Turing** (1912–1954), whose work with the British Government Code and Cypher School at Bletchley Park in the breaking of the German Enigma code was of such importance that his biographer (Hodges 1983) does not hesitate to deem him "the most important single person in winning the war" (an epithet certainly applied to a number of people). Turing was in a way well prepared for the task: his mathematical research was centered on an abstract "machine" designed to cope with any possible computational task, while cracking the Enigma code was based on clever electronic devices checking the captured code sequences. The Germans' initial reaction upon realizing that their information had been deciphered was similar to that of Philip II toward Viète: they suspected espionage and betrayal among their own troops.

Less known than Turing's work is that performed by one of the most brilliant mathematicians in Sweden, **Arne Beurling** (1905–1986), on the German G-Schreiber code in 1940: in two weeks he cracked the code and gave Sweden complete information on Germany's maneuvers in Norway (Beckman 2002).

4.3. Computers

The paths leading to the now ubiquitous digital computer are numerous, but many of them touch mathematics and war. The first real electronic computer, the American ENIAC, was constructed in hopes of computing artillery firing tables for use in World War II. The construction was not ready before the end of the war, but the machine actually was used for war purposes, in preparation of the hydrogen bomb. The German **Konrad Zuse**'s (1910–1995) work on electro-mechanical computers began as private experimentation before the war but was utilized by the German aviation industry (Zuse's plans to develop his computer into an electronic one were dropped because Germany was just about to win the war, so the computers were not needed anymore). The British had a huge special-purpose electronic computer, dubbed the Colossus, in Bletchley Park, working for cryptological purposes. It is noteworthy that the two mathematicians with perhaps the greatest influence on the course of World War II, Turing and von Neumann, had strong intellectual input in the development of the computer. Independently of each other they arrived at the crucial idea of an all-purpose computer, one that stores the program electronically in its memory.

5. Conclusion

These examples from history do not suffice to establish the existence of a specific link joining two human activities, one very abstract and distant from the world as perceived by the majority of people, the other very earthbound and often associated with rather undesirable traits of the human race. But they suffice to establish the historical fact that the two fields have to some extent been of use to each other. Also, the examples show that in several cases the useful—in our case, mathematical—tool was created long before its usefulness was recognized.

For mathematicians, teaching often provides the income without which even a scientist is unable to exist. For many of the mathematicians named in this article as well as numerous others, teaching in military institutions provided the material means for creative work. In many cases, the military has received in exchange the information this teaching has conveyed. In rarer cases, the mathematician has been able to give something more substantial. That these cases are not so common should not be a surprise. In any pursuit, exceptional results are uncommon, and they are not produced on demand. But if you close all opportunities you never obtain them.

To excel in war, one needs superior force and superior intellect. The latter in particular is impossible to produce on demand. To have at least the option of utilizing novel ideas unknown to the adversary, it might be advisable to maintain a reserve of potentially capable people. One should not demand results and usefulness on a daily or monthly basis and should be content with the fact that no results may ever come forth. But—especially for a small nation that cannot build a force physically matching that of a stronger potential enemy—the hope of superiority lies in intellectual superiority. Herein may lie one justification for employing high-quality teachers in military schools.

Sources

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